

TEMA II b) EDO(2) LCC NH

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = Q(x).$$

$$Q(x) \neq 0$$

- a) Método del Operador Diferencial
- b) Método de Parámetros Variables.

$$\boxed{\begin{aligned} (D - a)[y(x)] &= 0 \\ \frac{dy}{dx} - ay &= 0 \\ y' - ay &= 0 \\ \dot{y} - ay &= 0 \end{aligned}}$$

$$(D^2 + a_1 D + a_2)[y(x)] = 0$$

$$\rightarrow m^2 + a_1 m + a_2 = 0$$

$$(m - m_1)(m - m_2) = 0 \quad m_1 \neq m_2 \in \mathbb{R} \quad \text{CASO I.}$$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$\rightarrow (D - m_1)(D - m_2)[c_1 e^{m_1 x} + c_2 e^{m_2 x}] = 0$$

$$0 \equiv 0$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$y_g = C_1 e^{3x} + C_2 e^{2x}$$

$$(D^2 - 5D + 6)[y(x)] = 0$$

$$(D - 3)(D - 2)[y(x)] = 0$$

$$(D - 3)(D - 2)[C_1 e^{3x} + C_2 e^{2x}] = 0$$

$$(D - 3) \left[3C_1 e^{3x} + 2C_2 e^{2x} - 2C_1 e^{3x} - 2C_2 e^{2x} \right] = 0$$

$$(D - 3)[C_1 e^{3x}] = 0$$

$$3C_1 e^{3x} - 3C_1 e^{3x} = 0$$

$$0 \equiv 0$$

ce^{ax} Cxe^{ax} $Cx^n e^{ax}$	$(D-a)$ $(D-a)^2$ $(D-a)^{n+1}$
C_1 Cx Cx^n	D D^2 D^{n+1}
$\cos(bx)$ $\text{sen}(bx)$	$\} (D^2 + b^2)$
$e^{ax} \cos(bx)$ $e^{ax} \text{sen}(bx)$	$\} ((D-a)^2 + b^2)$
$xe^{ax} \cos(bx)$ $x^p e^{ax} \text{sen}(bx)$	$\} ((D-a)^2 + b^2)^2$
$x^n e^{ax} \cos(bx)$ $x^n e^{ax} \text{sen}(bx)$	$\} ((D-a)^2 + b^2)^{n+1}$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 4e^{2x} - x^2 \quad \text{EDO(2) LCCNH}$$

$$(D^2 - 5D + 6)y = 0$$

$$(D-3)(D-2)y = 0$$

$$y = C_1 e^{3x} + C_2 e^{2x}$$

$$(D-3)(D-2)(D-2)(D)^3 y = 0$$

$$(D-3)(D-2)^2 D^3 y = 0$$

$$y_g = C_1 e^{3x} + C_2 e^{2x} + C_3 x e^{2x} + C_4 + C_5 x + C_6 x^2$$

$$y = C_1 e^{3x} + C_2 e^{2x} + A x e^{2x} + B + D x + E x^2$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 4e^{2x} - x^2$$

$$y_{p/a} = A x e^{2x} + B + D x + E x^2$$

$$\frac{dy}{dx} = A(2x e^{2x} + e^{2x}) + (0) + D + 2E x$$

$$\frac{d^2 y}{dx^2} = A(2(2x e^{2x} + e^{2x}) + 2e^{2x}) + (0) + 2E$$

$$\frac{d^2 y}{dx^2} = A(4x e^{2x} + 4e^{2x}) + 2E$$

$$5 \frac{dy}{dx} = -A(10x e^{2x} + 5e^{2x}) - 5D - 10E x$$

$$+ 6y = 6A x e^{2x} + 6B + 6D x + 6E x^2$$

$$4e^{2x} = (0)x e^{2x} - A e^{2x} \quad \boxed{A = -4}$$

$$-x^2 = (6B - 5D + 2E) + (6D - 10E)x + 6E x^2$$

$$6E = -1 \quad E = -\frac{1}{6}$$

$$6D - 10E = 0 \quad 6D = -\frac{10}{6} \Rightarrow D = -\frac{10}{36}$$

$$6B - 5D + 2E = 0$$

$$6B = 5D - 2E \quad 6B = -\frac{50}{36} + \frac{1}{3} \Rightarrow B = -\frac{38}{36}$$

$$y_g = C_1 e^{3x} + C_2 e^{2x} - 4x e^{2x} - \frac{38}{36} - \frac{10}{36} x - \frac{1}{6} x^2$$

$$\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 5e^{3x} + \cos(2x) + 3x$$

$$(D^3 + D^2 + D)y = 0$$

$$(D^2 + D + 1)Dy = 0$$

$$m^2 + m + 1 = 0 \quad m = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$\left((D - \frac{1}{2}) \pm (\frac{\sqrt{3}}{2}i)\right)Dy = 0 \quad m = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\left((D + \frac{1}{2})^2 + \frac{3}{4}\right)Dy = 0$$

$$(D^2 + D + 1)Dy = 5e^{3x} + \cos(2x) + 3x$$

$$(D^2 + D + 1)D(D-3)(D^2+4)D^2y = 0$$

$$y_g = C_1 e^{-\frac{1}{2}x} \cos(\frac{\sqrt{3}}{2}x) + C_2 e^{-\frac{1}{2}x} \sin(\frac{\sqrt{3}}{2}x) + C_3 + C_4 e^{3x} + C_5 \cos(2x) +$$

$$+ C_6 \sin(2x) + C_7 x + C_8 x^2$$

$$y_p = A e^{3x} + B \cos(2x) + D \sin(2x) + E x + F x^2$$

$$\frac{dy}{dx} = 3A e^{3x} - 2B \sin(2x) + 2D \cos(2x) + E + 2Fx$$

$$\frac{d^2 y}{dx^2} = 9A e^{3x} - 4B \cos(2x) - 4D \sin(2x) + (0) + 2F$$

$$\frac{d^3 y}{dx^3} = 27A e^{3x} + 8B \sin(2x) - 8D \cos(2x) + (0)$$

$$= 5e^{3x}$$

$$= 39A e^{3x}$$

$$A = \frac{5}{39}$$

$$\cos(2x) = (6B - 4D) \sin(2x) + (-4B - 6D) \cos(2x)$$

$$= (E + 2F) + 2Fx$$

$$E + 2F = 0$$

$$2F = 3$$

$$\begin{aligned} -4B - 6D &= 1 \\ 6B - 4D &= 0 \end{aligned}$$

$$\text{Tipo III.} - m^2 + a_1 m + a_2 = 0$$

$$m_1 = a + bi$$

$$m_2 = a - bi$$

$$y_g = e^{ax} (C_1 \cos(bx) + C_2 \operatorname{sen}(bx))$$

$$(m - (a + bi))(m - (a - bi)) = 0$$

$$((m - a) + bi)((m - a) - bi) = 0$$

$$((m - a)^2 + b^2) = 0$$